Name:

1. (a) (1 point) Flip three fair coins. What is the probability that you get exactly two heads?

$\frac{3}{8}$

   (b) (2 points) Let X denote the number of heads in three flips of a fair coin. Write down the pmf for X.

$$f(0) = \frac{1}{8}$$
$$f(1) = \frac{3}{8}$$
$$f(2) = \frac{3}{8}$$
$$f(3) = \frac{1}{8}$$

   (c) (2 points) X as in part (b). What are $E(X)$ and $\text{Var}(X)$?

$$E(X) = 0 \cdot \frac{1}{8} + 1 \cdot \frac{3}{8} + 2 \cdot \frac{3}{8} + 3 \cdot \frac{1}{8} = \frac{3}{2}$$
$$E(X^2) = 0^2 \cdot \frac{1}{8} + 1^2 \cdot \frac{3}{8} + 2^2 \cdot \frac{3}{8} + 3^2 \cdot \frac{1}{8} = 3$$
$$\text{Var}(X) = E(X^2) - (E(X))^2 = 3 - \frac{9}{4} = \frac{3}{4}.$$

2. (1 point) Suppose the pmf for a random varible X is given by

$$f(k) = \begin{cases} \frac{2}{3} \left(\frac{1}{3}\right)^k & k = 0, 1, 2, 3, \cdots \\ 0 & \text{otherwise} \end{cases}.$$

Write down a series that computes $E(X)$.

$$\sum_{k=0}^{\infty} k \frac{2}{3} \left(\frac{1}{3}\right)^k$$

3. (1 point) Let X denote the number of times lightning strikes the Empire State Building in a year. Suppose X follows a Poisson distribution with $\lambda = 25$. What is the probability that lightning strikes the building at least twice during the year?

$$P(X \geq 2) = 1 - P(X < 2) = 1 - \frac{25^0 e^{-25}}{0!} - \frac{25^1 e^{-25}}{1!} = 1 - 26e^{-25}.$$

4. Consider a random variable X whose pdf is

$$f(x) = \begin{cases} \frac{1}{2}\sin(x) & x \in [0, \pi] \\ 0 & \text{otherwise} \end{cases}.$$

(a) (1 point) What is $P(X \geq \frac{\pi}{6})$?

$$\int_{\pi/6}^{\pi} \frac{1}{2}\sin(x)dx = \frac{1}{2}\left(1 + \frac{\sqrt{3}}{2}\right)$$

(b) (2 points) What is $E(X)$?

$$\int_0^{\pi} x \frac{1}{2}\sin(x)dx = \frac{\pi}{2}$$

(integration by parts)